

NATURAL LOGIC

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Background

Extended Syllogistic Inference

Logics with individual variables, $RCA^\dagger(opp)$

Inference with Monotonicity and Polarity

Conclusion & Summary

Backup Slides

Background

- Commonsense Reasoning, Ordinary Reasoning
 - Marvin Minsky: For generations scientists and philosopher have tried to explain ordinary reasoning in terms of logical principles-with virtually no success. I suspect this enterprise failed because it was looking the wrong direction: common sense works so well not because it is an approximation of logic; logic is only a small part of our great accumulation of different, useful ways to chain things together. (cf. Minsky, 1985, 187)
 - Larry Moss: We should not be afraid of doing logic beyond logic. Joining the perspectives of **semantics, complexity theory, proof theory, cognitive science, and computational linguistics** should allow us to ask interesting questions and answer them. (cf. Moss, Berkeley, 2015, slides)

- Textual Entailment

- t entails h ($t \Rightarrow h$) if, typically, a human reading t would infer that h is **most likely** true.
- text: If you help the needy, God will reward you.
hypothesis: Giving money to a poor man has good consequences.
(cf. wikipedia, textual entailment)
- Angeli G, Manning CD. NaturalLI: Natural Logic Inference for Common Sense Reasoning[C]//Conference on Empirical Methods in Natural Language Processing. 2014:534-545.

..... inference in language is one of the main motivations for logic. However, in most presentations this motivation does not last long. I suspect that this is because the actual goals of the presentations are the connections to mathematics and/or computer science, and so the connections to natural language inference are quickly set aside. (Moss, 2015)

Semantics & Entailment Relations

- Semantics is primarily the linguistic, and also philosophical, study of meaning—in language, programming languages, formal logics, and semiotics. It focuses on the relationship between signifiers—like words, phrases, signs, and symbols—and what they stand for, their denotation. (cf. Wikipedia, semantics)
- What is the overall motivation for the field of semantics?
The received view is that the goal of the enterprise is to study entailment relations (or other related relations). That is, one considers intuitive judgments of entailment and nonentailment, and then attempts to build some sort of theory that accounts for those judgments. (cf. Moss, 2015)

The model-theoretic entailment relations

- One defines **truth conditions** for sentences to hold in some class of models,
- and then formulates a notion of **semantic consequence**: a sentence φ is a semantic consequence of a set of sentences Γ if every model of all sentences in Γ is also a model of φ . (cf. Moss, 2015)

What is natural logic?

- Logic for natural language, Logic in natural language
- Johan van Benthem: ‘Natural Logic’ is a somewhat loose, but popular and suggestive term for recurrent attempts over the last decades at describing basic patterns of human reasoning directly in natural language without the intermediate of some formal system. (cf. van Benthem, 2008)
- Maarten Maartensz: A collection of terms and rules that come with Natural Language that allows us to reason and argue in it. ¹
- Larry Moss: There is not exactly a well established field of natural logic, but we would like to think that handbook chapters like this one might solidify interest in the topic. (cf. Moss, 2015)

¹(cf. <http://www.maartensz.org/philosophy/Dictionary/N/Natural%20Logic.htm>)

The main study contents of Natural Logic

- Extended Syllogistic Inference
- Logic with Individual Variables
- Inference with Monotonicity and Polarity

Conservative & radical subprograms

The study in natural logic can be divided into two groups, which we might call the “conservative” and “radical” subprograms.

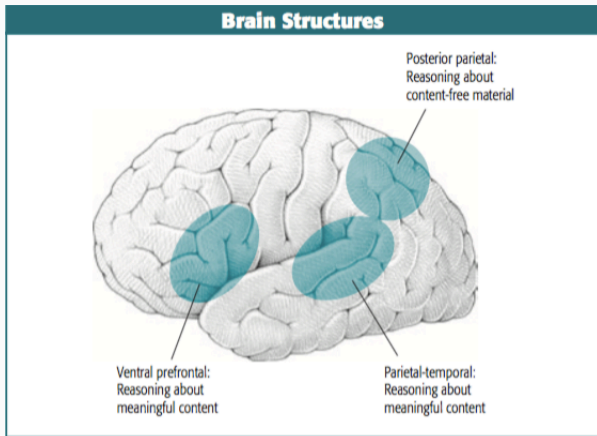
- The conservative program is to expand the syllogistic systems, but to continue to deal with extensional fragments of language.
- The more radical proposal explores the possibility of having proof theory as the mathematical underpinning for semantics in the first place. This view is suggested in the literature on philosophy of language, but it is not well explored in linguistic semantics because formal semantics is currently focussed on model-theoretic semantics.

Examples of inferences

- (1)
$$\frac{\text{Some dog sees some cat}}{\text{Some cat is seen by some dog}}$$
- (2)
$$\frac{\text{Bao is seen and heard by every student} \quad \text{Amina is a student}}{\text{Amina sees Bao}}$$
- (3)
$$\frac{\text{All skunks are mammals}}{\text{All who fear all who respect all skunks fear all who respect all mammals}}$$
- (4)
$$\frac{\begin{array}{l} \text{Every giraffe is taller than every gnu} \\ \text{Some gnu is taller than every lion} \\ \text{Some lion is taller than some zebra} \end{array}}{\text{Every giraffe is taller than some zebra}}$$
- (5)
$$\frac{\text{More students than professors run} \quad \text{More professors than deans run}}{\text{More students than deans run}}$$

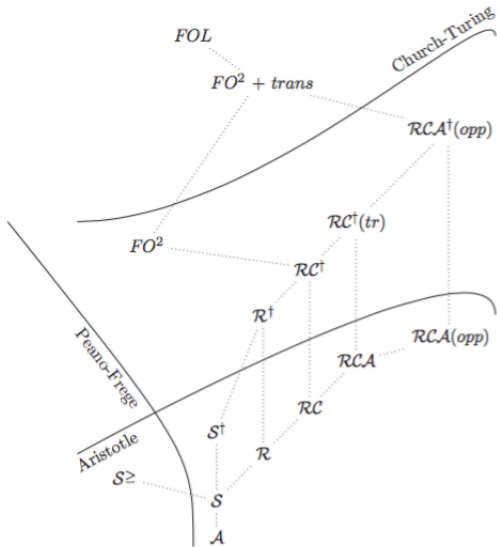
Why not use first-order logic?

- Example inference (5) is not expressible in first-order logic.
- Decidability
 - It is well known that first-order logic is undecidable: there is no algorithm that, given a finite set Γ of sentences and another sentence φ tells if φ is a semantic consequence of Γ .
 - This fundamental result is known as Church-Turing Thesis.
 - the intuitive concept of algorithm \iff the algorithm of Turing machine



Extended Syllogistic Inference

The spectrum of natural logic



first-order logic

$FO^2 + \text{"R is trans"}$

$FO^2 = 2 \text{ variable FOL}$

\dagger adds full N -negation

$RC(tr) + \text{opposites}$

$RC + (transitive)$

comparative adjs

$RC = \mathcal{R} + \text{relative clauses}$

$\mathcal{S} + \text{full } N\text{-negation}$

$\mathcal{R} = \text{relational syllogistic}$

\mathcal{S}^{\geq} adds $|p| \geq |q|$

\mathcal{S} : all/some/no p are q

\mathcal{A} : all p are q

Three kinds of divisions

- The line marked “Aristotle” separates the logical systems below the line, systems which can be profitably studied on their own terms without devices like variables over individuals, from those which cannot.
- The line we called “Church-Turing” is the division between systems that are decidable and those which are not.
- The line marked “Peano-Frege” is the division between systems that are expressible in first-order language and those which are not.

- The smallest system in the chart is \mathcal{A} , a system whose sentences are All p are q where p and q are variables.
- The next smallest system in the chart is \mathcal{S} , a system even smaller than the classical syllogistic. It adds sentences Some p are q to \mathcal{A} .
- \mathcal{S}^{\geq} adds additional sentences of the form “there are at least as many p as q ”.
- The language \mathcal{S}^{\dagger} adds full negation on nouns to \mathcal{S} . For example, one can say All p are q with the intended reading “no p are q .”

- The system \mathcal{R} extends § by adding verbs, interpreted as arbitrary relations. So \mathcal{R} would contain “Some dogs chase no cats”.
- The larger system \mathcal{RC} would contain relative clauses as exemplified in “All who love all animals love all cats”.
- $\mathcal{R}^\dagger, \mathcal{RC}^\dagger$ allow subject nouns to be negated. E.g. Every non-dog runs. It is unnatural in the standard speech.
- $\mathcal{RC}(tr), \mathcal{RC}^\dagger(tr)$ are extended from $\mathcal{RC}, \mathcal{RC}^\dagger$ by adding the **comparative adjective phrases** respectively.

The smallest system, \mathcal{A}

cf. Larry Moss's slides, Berkeley, 2015

THE SIMPLEST FRAGMENT “OF ALL”

Syntax: Start with a collection of **nouns**.
Then the **sentences** are the expressions

All p are q

Semantics: A **model** \mathcal{M} is a set M ,
together with an interpretation $\llbracket p \rrbracket \subseteq M$ for each noun p .

$$\mathcal{M} \models \textit{All } p \textit{ are } q \quad \text{iff} \quad \llbracket p \rrbracket \subseteq \llbracket q \rrbracket$$

cf. Larry Moss's slides, Berkeley, 2015

THE SEMANTICS IS TRIVIAL, AS IT SHOULD BE

Let $M = \{1, 2, 3, 4, 5, 6, 7, 8\}$.

Let $\llbracket a \rrbracket = \{1, 2, 3, 4, 5, 6\}$.

Let $\llbracket x \rrbracket = \{1, 4\}$.

Let $\llbracket y \rrbracket = \{2, 4\}$.

$\mathcal{M} \models \text{All } x \text{ are } a$

$\mathcal{M} \not\models \text{All } a \text{ are } x$

$\mathcal{M} \not\models \text{All } y \text{ are } x$

$\mathcal{M} \models \text{All } y \text{ are } a$

$\mathcal{M} \models \text{All } a \text{ are } a$

cf. Larry Moss's slides, Berkeley, 2015

SEMANTIC AND PROOF-THEORETIC NOTIONS

If Γ is a set of sentences, we write $\mathcal{M} \models \Gamma$ if for all $\varphi \in \Gamma$, $\mathcal{M} \models \varphi$.

$\Gamma \models \varphi$ means that every $\mathcal{M} \models \Gamma$ also has $\mathcal{M} \models \varphi$.

All of this is **semantic**.

cf. Larry Moss's slides, Berkeley, 2015

The rules are

$$\frac{}{All\ p\ are\ p} \qquad \frac{All\ p\ are\ n \quad All\ n\ are\ q}{All\ p\ are\ q}$$

A **proof tree over Γ** is a finite tree \mathcal{T} whose nodes are labeled with sentences, and each node is either an element of Γ , or comes from its parent(s) by an application of one of the rules.

$\Gamma \vdash \varphi$ means that there is a proof tree \mathcal{T} for over Γ whose root is labeled φ .

cf. Larry Moss's slides, Berkeley, 2015

Let Γ be the set

$\{All\ a\ are\ b, All\ q\ are\ a, All\ b\ are\ d, All\ c\ are\ d, All\ a\ are\ q\}$

Let φ be *All q are d*.

Here is a proof tree showing that $\Gamma \vdash \varphi$:

$$\frac{All\ q\ are\ a \quad \frac{All\ a\ are\ b \quad All\ b\ are\ d}{All\ a\ are\ d} B}{All\ q\ are\ d} B$$

Completeness of \mathcal{A}

- Let Γ be a set of sentences in any fragment containing All. Define $u \leq_{\Gamma} v$ to mean that $\Gamma \models \text{All } u \text{ are } v$.
- This relation \leq_{Γ} is a preorder: it is reflexive and transitive relation on the set P of unary atoms. (It is not, in general, antisymmetric, so it is not, in general, a partial order.)
- Construct canonical model: $\mathcal{M} = (\mathbf{P}, \leq_{\Gamma}, \llbracket \cdot \rrbracket)$
 - \mathbf{P} is the set of unary atoms
 - \leq is defined as above
 - $\llbracket x \rrbracket = \{y \mid y \leq_{\Gamma} x\}$

Completeness of \mathcal{A}

- $\Gamma \models \varphi \implies \Gamma \vdash \varphi$
- Proof. Suppose $\Gamma \models \varphi$ and $\Gamma \not\vdash \varphi$. Then we try to show that $\mathcal{M} = (\mathbf{P}, \leq_{\Gamma}, \llbracket \cdot \rrbracket)$ satisfies Γ but not φ .
 - Obviously, $\mathcal{M} \models \Gamma$.
 - Suppose $\mathcal{M} \models \varphi$ and φ is *All pare* q . Then we have $\llbracket p \rrbracket \subseteq \llbracket q \rrbracket$. By the definition of $\llbracket \cdot \rrbracket$, we have $p \in \llbracket p \rrbracket$. Then $p \in \llbracket q \rrbracket$. Then $p \leq_{\Gamma} q$. Then $\Gamma \vdash \text{All pare } q$. Contradiction.

Logics with individual variables,
 $RCA^\dagger(opp)$

Logics with individual variables, $RCA^\dagger(opp)$

- Now we wish to consider a larger system, an extended syllogistic system, which we call $RCA^\dagger(opp)$. This system has transitive verbs, subject relative clauses, and it is capable of representing a fairly large class of natural language inferences.
- In fact, sentences in our sense do not have variable occurrences. But general sentences do include variables. They are only used in our proof theory.

Our intention is that

- unary atoms represent plural nouns,
- adjective atoms represent comparative adjective phrases such as larger than and smaller than,
- and tv atoms represent transitive verbs.
- We group the adjective atoms and tv atoms into binary atoms, and we use letters like r for those.
- Moving on, we have set terms; these are named because in the semantics they denote sets.
 - $\forall(\textit{boy}, \textit{girl})$ those who see all boys
 - $\exists(\textit{girl}, \textit{taller})$ those who are taller than some girl(s)
 - $\forall(\textit{boy}, \overline{\textit{see}})$ those who fail-to-see all boys
 - $\exists(\textit{girl}, \overline{\textit{see}})$ those who fail-to-see some girl

Logics with individual variables, $RCA^\dagger(opp)$

Table 18.2. Syntax of terms and sentences of $RCA^\dagger(opp)$.

Expression	Variables	Syntax
unary atom	p, q	
adjective atom	a	
tv atom	t	
binary atom	b	$a \mid t$
constant	k, j	
unary relational term	l, m	$p \mid \bar{l} \mid l \wedge m$
binary relational term	r, s	$b \mid r^{-1} \mid \bar{r} \mid r \wedge s$
set term	b, c, d	$l \mid \exists(c, r) \mid \forall(c, r)$
sentence	ϕ, ψ	$\forall(c, d) \mid \exists(c, d) \mid c(k) \mid r(k, j)$

Source: Moss (2012a).

A structure (for this language $\mathcal{RCA}^\dagger(opp)$) is a pair $\mathcal{M} = \langle M, \llbracket \cdot \rrbracket \rangle$, where M is a nonempty set, $\llbracket p \rrbracket \subseteq M$ for all $p \in P$, $\llbracket r \rrbracket \subseteq M^2$ for all $r \in R$, and $\llbracket k \rrbracket \in M$ for all $k \in K$. That is, models now come with the semantics of the constants.

- $\llbracket \bar{l} \rrbracket = M - \llbracket l \rrbracket$
- $\llbracket l \wedge m \rrbracket = \llbracket l \rrbracket \cap \llbracket m \rrbracket$
- $\llbracket \bar{r} \rrbracket = M^2 - \llbracket r \rrbracket$
- $\llbracket r^{-1} \rrbracket = \llbracket r \rrbracket^{-1}$
- $\llbracket r \wedge s \rrbracket = \llbracket r \rrbracket \cap \llbracket s \rrbracket$
- $\llbracket \exists(l, t) \rrbracket = \{x \in M : \text{for some } y \in \llbracket l \rrbracket, \llbracket t \rrbracket(x, y)\}$
- $\llbracket \forall(l, t) \rrbracket = \{x \in M : \text{for all } y \in \llbracket l \rrbracket, \llbracket t \rrbracket(x, y)\}$

- $\mathcal{M} \models \forall(c, d) \iff \llbracket c \rrbracket \subseteq \llbracket d \rrbracket$
- $\mathcal{M} \models \exists(c, d) \iff \llbracket c \rrbracket \cap \llbracket d \rrbracket \neq \emptyset$
- $\mathcal{M} \models c(k) \iff \llbracket c \rrbracket(\llbracket k \rrbracket)$
- $\mathcal{M} \models r(k, j) \iff \llbracket r \rrbracket(\llbracket k \rrbracket, \llbracket j \rrbracket)$
- If Γ is a set of formulas, we write $\mathcal{M} \models \Gamma$ if for all $\varphi \in \Gamma, \mathcal{M} \models \varphi$.
- Satisfiable: A sentence φ is satisfiable if there exists \mathcal{M} such that $\mathcal{M} \models \varphi$
- Semantic Consequence $\Gamma \models \varphi$: every model of every sentence in Γ is also a model of φ .

Some dog sees some cat.

Some cat is seen by some dog

$$\frac{\exists(\text{dog}, \exists(\text{cat}, \text{see}))}{\exists(\text{cat}, \exists(\text{dog}, \text{see}^{-1}))}$$

Proof System

$$\frac{c(t) \quad \forall(c,d)}{d(t)} \forall E$$

$$\frac{c(t) \quad d(t)}{\exists(c,d)} \exists I$$

$$\frac{\begin{array}{c} [c(x)] \\ \vdots \\ d(x) \end{array}}{\forall(c,d)} \forall I$$

$$\frac{c(u) \quad \forall(c,r)(t)}{r(t,u)} \forall E$$

$$\frac{r(t,u) \quad c(u)}{\exists(c,r)(t)} \exists I$$

$$\frac{\begin{array}{c} [c(x)] \\ \vdots \\ r(t,x) \end{array}}{\forall(c,r)(t)} \forall I$$

Proof System

$$\frac{\exists(c, d) \quad \begin{array}{c} [c(x)] \quad [d(x)] \\ \vdots \\ \alpha \end{array}}{\alpha} \exists E$$

$$\frac{\exists(c, r)(t) \quad \begin{array}{c} [c(x)] \quad [r(t, x)] \\ \vdots \\ \alpha \end{array}}{\alpha} \exists E$$

$$\frac{r(k, j) \quad s(k, j)}{(r \wedge s)(k, j)} \wedge \quad \frac{r^{-1}(j, k)}{r(k, j)} \text{inv}$$

$$\frac{a(k, j) \quad a(j, \ell)}{a(k, \ell)} \text{trans}$$

$$\frac{\alpha \quad \bar{\alpha}}{\perp} \perp I$$

$$\frac{\begin{array}{c} [\bar{\phi}] \\ \vdots \\ \perp \end{array}}{\phi} \text{RAA}$$

Inference with Monotonicity and Polarity

cf. Larry Moss's slides, Berkeley, 2015

*"The star-belly Sneetches have bellies with stars;
the plain-belly Sneetches have none upon thars"*



cf. Larry Moss's slides, Berkeley, 2015



animal
Sneetch
Star-Belly Sneetch



move
dance
waltz

Let's talk about a situation where

all Sneetches dance.

Which one would be true?

- ▶ all Star-Belly Sneetches dance
- ▶ all animals dance

cf. Larry Moss's slides, Berkeley, 2015



animal
Sneetch
Star-Belly Sneetch



move
dance
waltz

Let's put the arrows on the words **Sneetches** and **dance**.

- ① No Sneetches[↓] dance[↓].
- ② If you play loud enough music, any Sneetch[↓] will dance[↑].
- ③ Any Sneetch[↓] in Zargonía would prefer to live in Yabistan.
- ④ If any Sneetch[↓] dances[↓], McBean will dance[↑], too.

cf. Larry Moss's slides, Berkeley, 2015

WHAT GOES UP? WHAT GOES DOWN?

$$f(x^{\downarrow}, y^{\uparrow}) = y - x \quad (1)$$

$$g(x^{\uparrow}, y^{\downarrow}) = x + \frac{2}{y} \quad (2)$$

$$h(v^{\uparrow}, w^{\uparrow}, x^{\uparrow}, y^{\downarrow}, z^{\downarrow}) = \frac{x - y}{2^{z - (v + w)}} \quad (3)$$

The \uparrow and \downarrow notations have the same meaning in language as in math!

This is not an accident!

cf. Larry Moss's slides, Berkeley, 2015

FRACTIONS AND CANCELLING

$$\frac{7\cdot\cancel{8}\cdot 3}{8\cdot\cancel{8}\cdot 2} = \frac{21}{40}$$

$$\frac{\cancel{8}\cdot 5\cdot 3}{7\cdot 5\cdot\cancel{8}} = \frac{15}{40}$$

You can cancel down the middle.

You can cancel end-to-end.

$$\frac{7\cdot\cancel{4}\cdot 5}{\cancel{4}\cdot\cancel{4}\cdot 2} = \frac{35}{2}$$

But if you cancel wrongly, ...

cf. Larry Moss's slides, Berkeley, 2015

\ means "look left"
/ means "look right"

$$X \times (Y \backslash X) = Y$$

$$(X / Y) \times Y = X$$

cf. Larry Moss's slides, Berkeley, 2015

$$\begin{array}{c}
 \text{a : } NP/N \quad \text{Sneetch : } N \\
 \hline
 \text{teased: } (S \backslash NP)/NP \quad \text{a Sneetch: } NP \\
 \hline
 \text{McBean: } NP \quad \text{teased a Sneetch: } S \backslash NP \\
 \hline
 \text{McBean teased a Sneetch: } S
 \end{array}$$

$$\begin{array}{c}
 \text{criticized: } (S \backslash NP)/NP \quad \text{McBean: } NP \\
 \hline
 \text{criticized McBean: } S \backslash NP \quad \text{gently: } (S \backslash NP) \backslash (S \backslash NP) \\
 \hline
 \text{Seuss: } NP \quad \text{criticized McBean gently: } S \backslash NP \\
 \hline
 \text{Seuss criticized McBean gently: } S
 \end{array}$$

cf. Larry Moss's slides, Berkeley, 2015

TRADITIONAL ENGLISH SYNTAX AND DIRECTIONAL FRACTIONS

syntactic category	name in traditional grammar
S	sentence
N	noun
NP	noun phrase
N/N	adjective
$S \backslash NP$	verb phrase
$S \backslash NP$	intransitive verb
$(S \backslash NP) \backslash (S \backslash NP)$	adverb
$(S \backslash NP) / NP$	transitive verb
NP / N	determiner
$(N \backslash N) / (S \backslash NP)$	relative pronoun

cf. Larry Moss's slides, Berkeley, 2015

$$\begin{array}{c}
 \text{Sneetch} \leq \text{animal} \qquad \text{every} \leq \text{most} \\
 \hline
 \text{every animal} \downarrow \leq \text{every Sneetch} \downarrow \quad \text{every Sneetch} \downarrow \leq \text{most Sneetches} \downarrow \\
 \hline
 \text{every animal} \downarrow \leq \text{most Sneetches} \downarrow \\
 \hline
 \text{every animal} \downarrow \text{ hops} \uparrow \leq \text{most Sneetches} \downarrow \text{ move} \uparrow
 \end{array}$$

This is how a computer could do the reasoning:

if every animal hops
 then most Sneetches move

cf. Larry Moss's slides, Berkeley, 2015

Conclusion & Summary

Conclusion

- Semantics texts often say that the goal of the subject is to study inference the same way that syntax studies grammaticality. However, the actual work in semantics concerns the very challenging enterprise of defining the semantics, that is giving the truth conditions. It is much more common to see proposals and models justified by making sure that unintuitive consequences do not formally obtain than to be sure that intuitive ones do indeed hold.
- The main methodological point in our study is to make inference the main goal. This means that we have so far restricted our study to fragments much smaller than those studied in semantics because with smaller fragments one can propose logics more or less uncontroversially, and then study them to get a fuller account of inference.

Summary

- Background
- The smallest system \mathcal{A}
- The larger system $\mathcal{RCA}^\dagger(opp)$
- Inference with Monotonicity and Polarity
- Conclusion.

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Thank you!

Backup Slides

The border line between traditional and modern logic

- Indeed, I would claim that, for the purpose of analyzing ordinary human inference, the modern 'first-order/higher-order' boundary is mainly a mathematical 'systems concern' without any clear matching jump in natural reasoning.
- Traditional logic investigated these phenomena for a wide range of expressions, without any boundary between unary and binary predication –another artefact of viewing history through predicate-logical glasses.

Major difficulty of the traditional logic

- Traditional logic had a major difficulty: providing a systematic account of complex linguistic constructions from which to infer, and in particular, despite lots of valid insights, it wrestled with a good general account of iterations of quantifiers.
- Dummett 1973 makes a lot of this, by saying that Frege' s compositional treatment in terms of merely explaining single quantifiers and then letting compositionality do all the rest “solved the problem which had baffled traditional logicians for millennia: just by ignoring it” . Again, while there is a kernel of truth to this, there is also a good deal of falsehood.
- Indeed, as the extensive historical study Sanchez 2004 remarks, it seems more fair to say that De Morgan represents a low point in logical history as far as understanding the scope of monotonicity reasoning is concerned. Things got better after him –but as the author points out tongue-in-cheek, they also got better and better moving back in time to Leibniz and then on to the Middle Ages...

- Until late in the 20th century, attempts have been made to further develop the Syllogistic into a full- fledged calculus of monotonicity reasoning, witness Sommers 1982, and Englebretsen 1981.
- The claim of these authors was that this enterprise provided a viable alternative to first-order logic for bringing out key structures in actual human reasoning in a more congenial way. Still they did not propose turning back the clock altogether.
- E.g., Sommers' book is up to modern standards in its style of development, providing a systematic account of syntactic forms, an arithmetical calculus for computing positive and negative syntactic occurrence, as well as further inferential schemata generalizing traditional inference patterns like Conversion and Contraposition.

The ‘natural logic’ program of 1980s

- In the 1980s, the idea arose that the preceding observations had a more general thrust. Natural language is not just a medium for saying and communicating things, but it also has a ‘natural logic’ , viz. a system of simple modules capturing ubiquitous forms of reasoning that can operate directly on natural language surface form without the usual logical formulas.
- This idea was developed in some detail in van Benthem 1986, 1987, whose main proposals we outline here. The main ingredients were to be three modules:
 - Monotonicity reasoning, or Predicate Replacement,
 - Conservativity, or Predicate Restriction, and also
 - Algebraic laws for inferential features of specific lexical items.

The challenge of natural logic

- There are many further natural subsystems in natural language, including reasoning about collective predication, prepositions, anaphora, tense and temporal perspective.
- The systematic challenge is then to see how much of all this inference can be done directly on natural language surface form, and we will look at some details below.
- Another challenge might be how these subsystems manage to work together harmoniously in one human mind, and we will return to this somewhat neglected issue below.

- Notice how this way of thinking cuts the cake of reasoning differently from the syntax of first-order logic –redrawing the border-line between traditional and modern logic.
- monotonicity inference is both richer and weaker than first-order predicate logic. It is weaker in that it only describes part of all valid quantifier inferences, but it is richer in that it is not tied to any particular logical system, as we observed above (it works for second-order just as well as first-order logic).

Thank you again!